Parameter Estimation of Centrality Metric for Dynamic Citation Networks

Jatuporn Leksut

August 28, 2010

Under the supervision of Professor Kristina Lerman
University of Southern California Information Sciences Institute

1 Introduction

Network analysis studies the structure of associations between nodes in a network. One important aspect of network analysis is to examine information flow and determine relative important nodes in a particular network. Googles PageRank [6] is a well-known example of how network analysis has advanced information science and technology. PageRank algorithm uses link-based centrality metric to identify nodes importance and thereby enable ranking of web documents. There exist a number of network analysis algorithms that are used for several purposes. However, network analysis algorithms mostly applied to static networks, while in reality most networks are dynamic since network topology usually changes over time due to the addition and removal of nodes. [5] has developed a centrality metric for dynamic networks to study the nature and information spread on dynamic networks. The authors of [**?**] has developed a novel centrality metric which considers the temporal order of edges based on *α-centrality metric*, defined by Bonacich[2], and [1]'s observation about the problem of time-aware ranking.

The unique feature of the dynamic centrality metric defined by [5] is that it takes time and length scale of interactions between nodes into account. Time and length scale of edges can be set through two parameters α and γ. These two parameters are adjustable. However, we analyzes data and applies algorithm to obtain appropriate values for these two parameters. Parameter α indicates the probability that the information will be transfered from one node to another. Parameter γ indicates the length of retention in which the node will remember the information it receives.

This project explores three citation network data sets and estimates α and γ parameters for each data set. The corresponding parameters will be used for performing ranking scientific papers in the given citation networks using dynamic centrality metric developed by [5].

2 Dynamic Centrality Metric

Given an adjacency matrix A that represents a network. A_{ij} indicates the number of edges between node i and node j. Bonacich[4] defines *α-centrality metric* as:

$$C^s(\alpha, \beta) = \beta A + \beta \alpha A \cdot A + \cdots + \beta \alpha^n A^{n+1} \cdots$$

where α and β are attenuation factors for indirect edge and direct edge respectively. α defines the length scale of interest in interactions between nodes. For α ¡ 1, the longer the path becomes, the less credit we acknowledge for that path. Starting from $\alpha = 0$ in which only direct path of length 1 is considered, as α increases, the scale of interest expands to cover longer paths.

Since the topology of a dynamic network changes over time, [5], following [3], defines an adjacency matrix $A(t_i)$ for each snapshot of a network at time $t_i (i \in 1, \ldots, n)$. In dynamic networks, future states of the network may depend on current and past states as past interactions are remembered by the nodes. [5] considered the trend that memory decays over time, and thereby introduced the *retained adjacency matrix* $R(t_n)$ at t_n:

$$R(t_n, \gamma) = A(t_n) + \gamma A(t_{n-1}) + \cdots + \gamma^{n-1} A(t_1)$$

Introduced by [5], the *retention probability* γ specifies the probability that a node retains information it receives at t_i until t_{i+1}. Knowing $R(t_n, \gamma)$, the *retained dynamic centrality matrix* can be derived as:

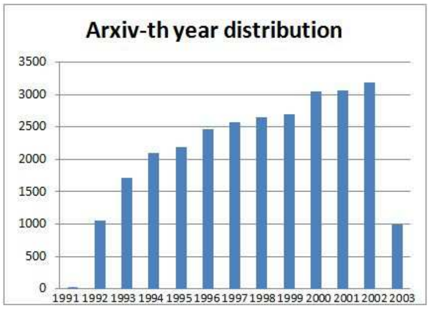
$$\begin{aligned} RC^d_{t_1 \to t_n}(\beta, \alpha, \gamma) =& \beta R(t_1, \gamma) + \beta \alpha R(t_1, \gamma) R(t_2, \gamma) + \beta \alpha^2 R(t_1, \gamma) R(t_2, \gamma) R(t_3, \gamma) \\ &+ \cdots + \beta \alpha^{n-1} R(t_1, \gamma) \cdots R(t_n, \gamma) \end{aligned}$$

3 Citation Networks

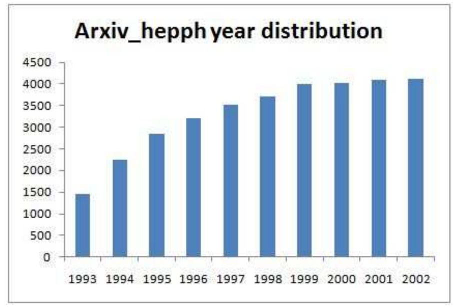
We studies three citation networks – Arxiv HEP-TH, Arxiv HEP-PH, and APS Physical Review. These networks form citation graphs where there is a directed edge from node i to node j if a paper i cites paper j. Each citation is represented by an edge which connected between two papers.

3.1 Arxiv HEP-TH

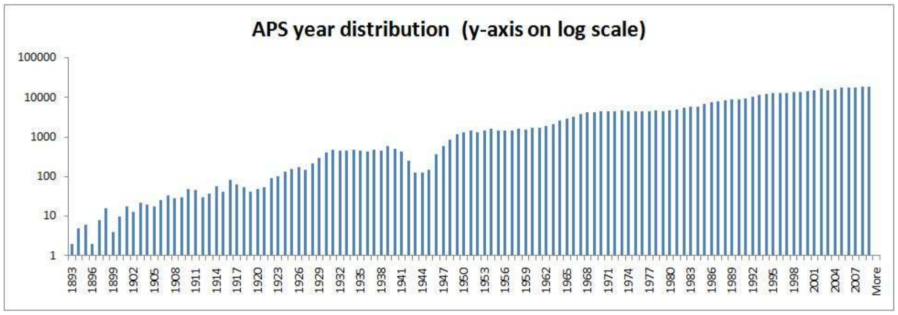
Arxiv HEP-TH(high energy physics theory) citation network [4] contains 351,025 citations and 27,733 papers from March 1991 to May 2003. However, the number of papers published



(a) Arxiv HEP-TH



(b) Arxiv HEP-PH



(c) APS Physical Review

Figure 1: Year distribution of articles from each data set

in 1991 and 2003 is relatively small, so we considered only 331,675 citations with 26,717 papers from 1992 - 2002.

3.2 Arxiv HEP-PH

Arxiv HEP-PH (high energy physics phenomenology) citation network [4] contains 421,578 citations and 34,546 papers from February 1992 February 2003. Note that each paper can have several dates of submission, so we decided to choose the first version of submission. However, it results in the negative age of a citation. There exist 2,972 citations that the citing papers appear to cite other papers in the future or papers that were published on the same date. Also, we decided to select only papers published in 1993 - 2002. Therefore, we take 393,565 citations with 33,229 papers from 1993 - 2002 into account.

3.3 APS Physical Review

APS Physical Review citation network contains 4,692,056 citations and 450,084 papers from year 1893 2009. Note that 4 out of 4,692,056 citations are not included in the estimation of parameters since 3 papers out of 450,084 papers do not have information about publication date provided. Therefore, we take 4,692,052 citations with 450,081 papers into account.

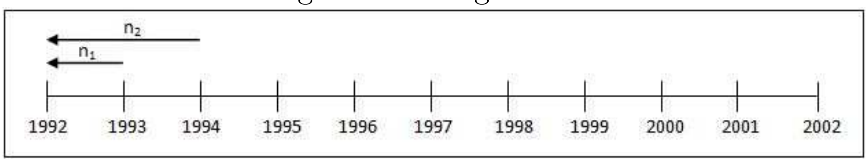
4 Parameter Estimation

4.1 estimation of γ

Parameter γ indicates the length of retention in which the node will remember the information it receives. The following section will demonstrate estimation of γ from Arxiv HEP-TH data set. To avoid the bias against longer interval, we only considered two kinds of citations: the ones that cite to the least recent year and the ones that cite from the most recent year which are 1992 and 2002 respectively.

4.1.1 Citations that cite to the least recent year

Figure 2: Citing to 1992

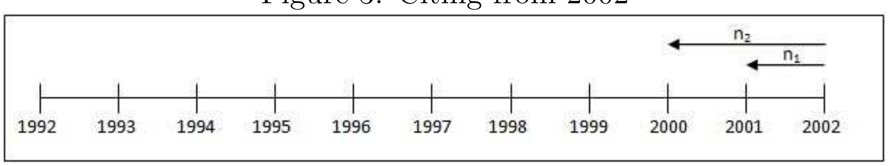


Assume t is the terminal year. Let n_i represents the number of citations that have paper published in year $t + i$ citing papers published in year t.

Define t to be 1992. n_1 then represents the number of papers published in 1993 citing papers published in 1992. n_2 represents the number of papers published in 1994 citing papers published in 1992 and so on.

4.1.2 Citations that cite from the most recent year

Figure 3: Citing from 2002



Assume t is the terminal year. Let n_i represents the number of citations that have paper published in year t citing papers published in year $t - i$.

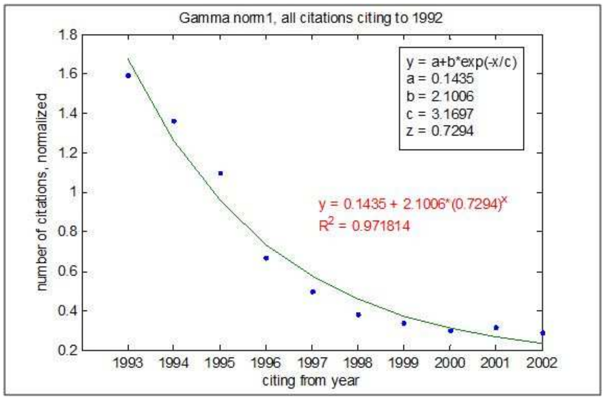
Define t to be 2002. n_1 then represents the number of papers published in 2002 citing papers published in 2001. n_2 represents the number of papers published in 2002 citing papers published in 2000 and so on.

We collected these data and see the distribution of citations over each time interval. In addition, we normalize n_i value by two options:

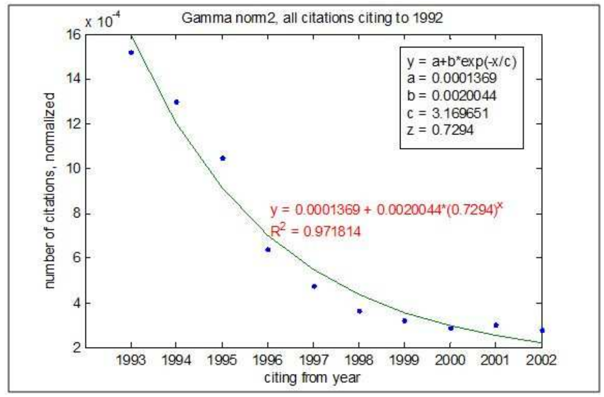
Options	Citing from t	Citing to t
Norm1	$n_{i,normalized} = \frac{n_i}{N_t}$	$n_{i,normalized} = \frac{n_i}{N_{t+1}}$
Norm2	$n_{i,normalized} = \frac{n_i}{N_t * N_{t-i}}$	$n_{i,normalized} = \frac{n_i}{N_t * N_{t+i}}$

Note that N_t is the number of paper published in year t

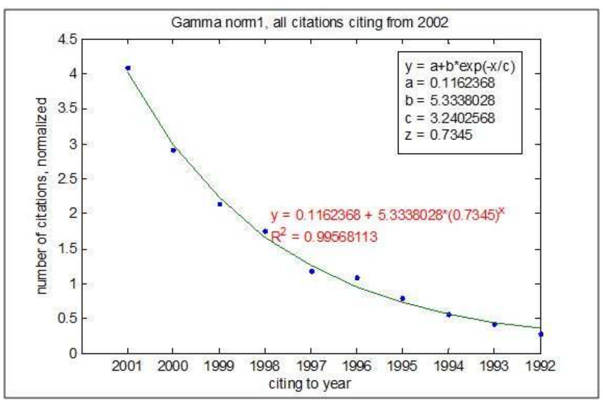
According to the previous study of Physical Review citation networks [7], [5] assumes that citation retention probability decays geometrically with time. Figure 4 shows the distribution for $n_{i,normalized}$ – normalized value of the number of papers over each time interval.



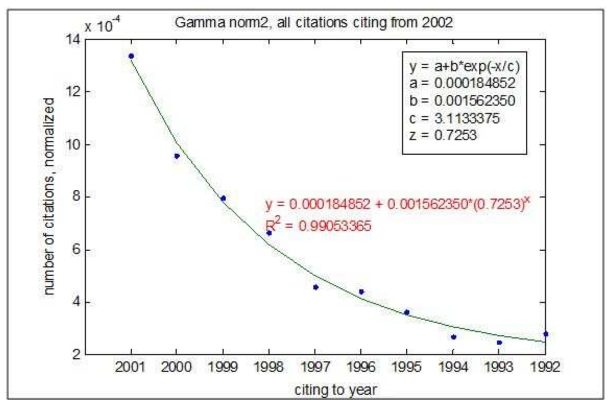
(a) γ-citing to 1992 Norm1



(b) γ-citing to 1992 Norm2



(c) γ-citing from 2002 Norm1



(d) γ-citing from 2002 Norm2

Figure 4: The distribution of papers over each time interval

The following table shows γ values obtained from *Arxiv HEP-TH*. See the plots from Figure 4 in which $y = a + b \cdot \gamma^x$.

	γ-Norm1	γ-Norm2
Citing to 1992	0.72943	0.72943
Citing from 2002	0.73446	0.72528

4.2 estimation of α

Parameter α indicates the probability that the information will be transfered from one node to another. The following section will demonstrate estimation of α from Arxiv HEP-TH data set.

Suppose A_j is the adjacency matrix containing citation links from papers published in year j citing to papers published in year $j-1$. For example, A_{2002} contains citation links from papers published in 2002 citing to papers published in 2001. a_{ij} represents a citation link of paper i (2001) cited by paper j (2002)

$$A_{2002} = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}$$

We find the distribution of citation chains that span consecutive year by performing matrix multiplication between adjacency matrices in consecutive year starting from 2002 to 1992. In the estimation of α, we also consider two methodology of analyzing citation chains. One is looking backwards starting from the most recent year i.e. 2002 in the example data set. The other one is looking forwards starting from the least recent year which is 1993 from *Arxiv HEP-TH* data set.

A matrix B_i represents a citation chains of length i. The following shows both methodology of determining α – forwards and backwards.

Backwards

$$\begin{aligned} B_1 &= A_{2002} \\ B_2 &= A_{2001}A_{2002} \\ B_3 &= A_{2000}A_{2001}A_{2002} \\ &\cdots \\ B_9 &= A_{1993}A_{1994}\cdots A_{2001}A_{2002} \end{aligned}$$

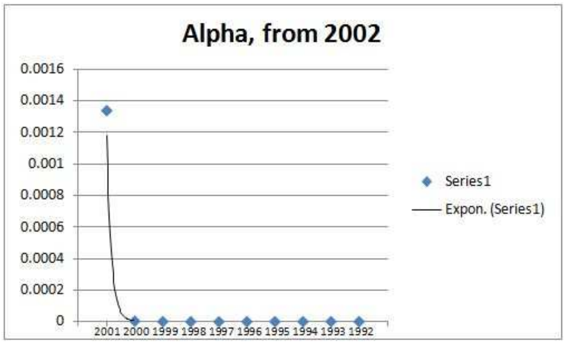
Forwards

$$\begin{aligned} B_1 &= A_{1993} \\ B_2 &= A_{1993}A_{1994} \\ B_3 &= A_{1993}A_{1994}A_{1995} \\ &\cdots \\ B_9 &= A_{1993}A_{1994}\cdots A_{2001}A_{2002} \end{aligned}$$

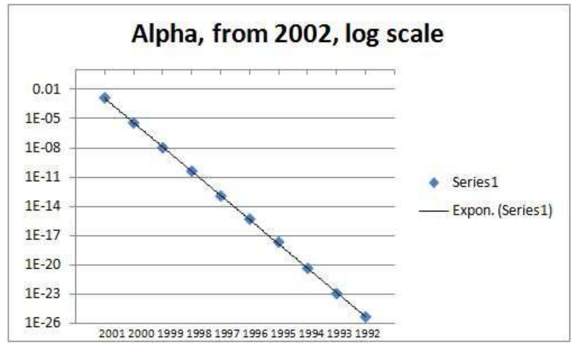
Let n_i be the number of chains with length i. To normalize it, we divide n_i by the product of the number of papers published in each year that the chains span over. N_t is the number of paper published in year t

$$n_{i,normalized} = \frac{n_i}{N_t N_{t+1} \cdots N_{t+i-1}}$$

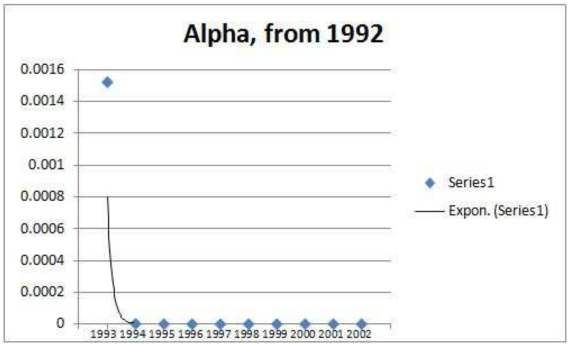
Again, [5] consider that the number of chains of length i decays geometrically with α. Figure 4 shows the distribution of the number of chains from *Arxiv HEP-TH* data set.



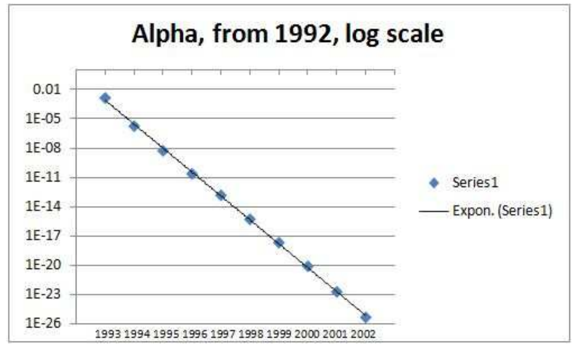
(a) α-Backwards from 2002



(b) α-Backwards on log scale



(c) α-Forwards from 1992



(d) α-Forwards on log scale

Figure 5: The distribution of the number of chains

The following table shows α values obtained from *Arxiv HEP-TH*. See the plots from Figure 5 in which $y = a + b \cdot \alpha^x$.

	α	$\frac{1}{1-\alpha}$
Backwards from 2002	0.007074	1.007124
Forwards from 1992	0.006313	1.006353

5 Results

5.1 Arxiv HEP-TH data set

5.1.1 γ values

The following table shows γ values obtained from *Arxiv HEP-TH*.

	γ-Norm1	γ-Norm2
Citing to 1992	0.72943	0.72943
Citing from 2002	0.73446	0.72528

5.1.2 α values

The following table shows α values obtained from *Arxiv HEP-TH*.

	α	$\frac{1}{1-\alpha}$
Backwards from 2002	0.007074	1.007124
Forwards from 1992	0.006313	1.006353

5.2 Arxiv HEP-PH data set

The following table shows γ values obtained from *Arxiv HEP-PH*.

	γ-Norm1	γ-Norm2
Citing to 1993	0.82588	0.82588
Citing from 2002	0.78932	0.63856

5.2.1 α values

The following table shows α values obtained from *Arxiv HEP-PH*.

	α	$\frac{1}{1-\alpha}$
Backwards from 2002	0.006361	1.006402
Forwards from 1993	0.006466	1.006508

5.3 APS Physical Review

Physical Review has covered more than 100 years of citation history. Due to the decrease in paper publications during WWI and WWII, we decided to partition data into several small subsets in which each subset contains consecutive 10 years of citation networks. See **Appendix** for additional details on γ and α values obtained from each Physical Review subset.

6 Conclusion

We obtain encouraging results for all three data sets. With α and γ values, we can analyze information spread and the structure of dynamic citation networks partitioned by year. The obtained parameters can be further used in ranking papers in given citation networks. In addition, we develop two methodologies for estimating α which will benefit the prediction of future ranking.

Acknowledgements

I would like to thank Professor Kristina Lerman and Rumi Ghosh for their wonderful mentorship and great inspiration, and thank DREU program for the amazing opportunity.

References

[1] R. Baeza-Yates, F. Saint-Jean, and C. Castillo. Web structure, dynamics and page quality. In A. H. F. Laender and A. L. Oliveira, editors, *String Processing and Information Retrieval*, volume 2476 of *Lecture Notes in Computer Science*, chapter 12, pages 453-461. Springer Berlin Heidelberg, Berlin, Heidelberg, September 2002.

[2] P. Bonacich. Eigenvector-like measures of centrality for assymetric relations. *Social Networks*, 23:191-201, 2001.

[3] D. Braha and Y. Bar-Yam. From centrality to temporary fame: Dynamic centrality in complex networks. *Social Science Research Network Working Paper Series*, 2006.

[4] J. Gehrke, P. Ginsparg, and J. M. Kleinberg. Overview of the 2003 KDD Cup. SIGKDD Explorations 5(2): 149-151, 2003.

[5] K. Lerman, R. Ghosh, and J.H. Kang. Centrality Metric for Dynamic Networks, *In MLG workshop at KDD*, 2010.

[6] L. Page, S. Brin, R. Motwani, and T. Winograd. The pagerank citation ranking: Bringing order to the web. Technical report, Stanford Digital Library Technologies Project, 1998.

[7] S. Redner. Citation statistics from 110 years of physical review. *Physics Today*, 58(6):49-54, 2005.

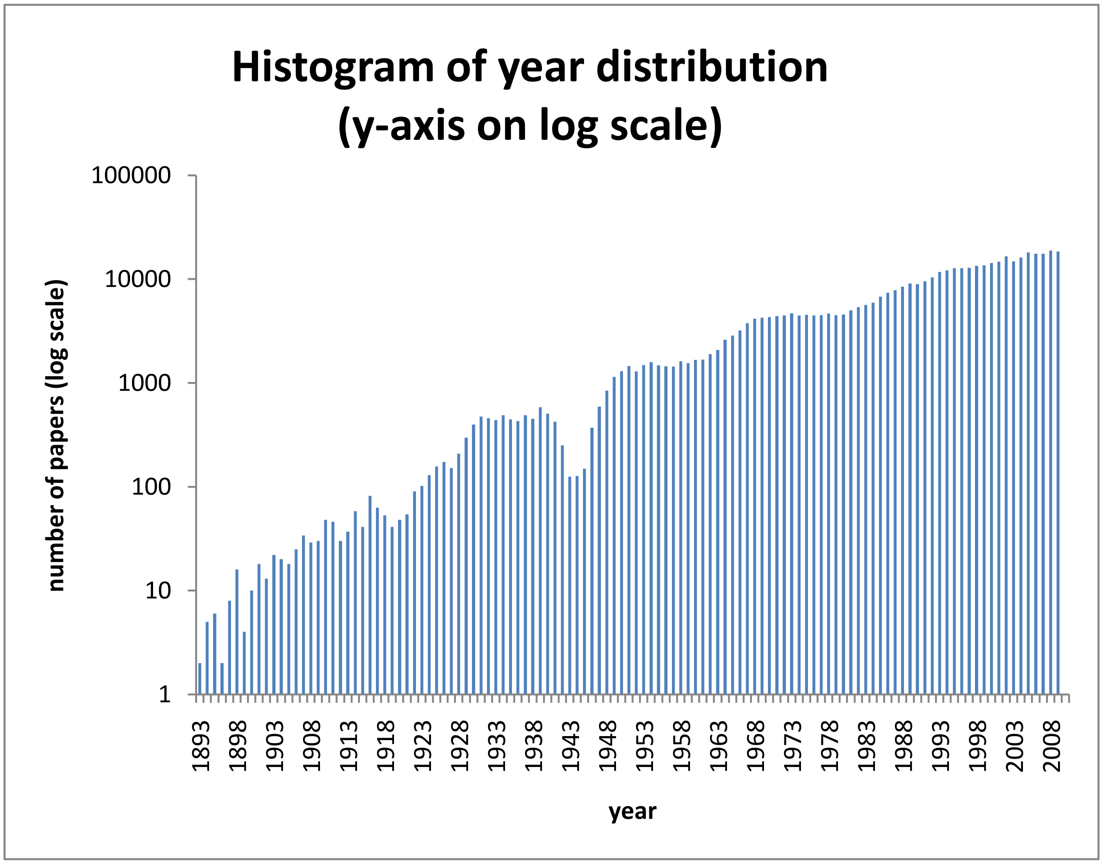
A Appendix

The following pages contain results from Physical Review data set.

Overview

APS data set

- There are 4,692,056 citations and 450,084 papers from year 1893 – 2009.
- Note that 4 out of 4,692,056 citations are not included in the calculations of parameter since 3 papers out of 450,084 papers do not have information about publication date provided.
- Therefore, to be precise, we take 4,692,052 citations with 450,081 papers into account.



Gamma estimation

Note that:

2009 - 2000 means papers published in 2009 citing to papers published in 2008, 2007, 2006, …, 2000

2000 - 2009 means papers published in 2000 are cited by papers published in 2001, 2002, 2003, …, 2009

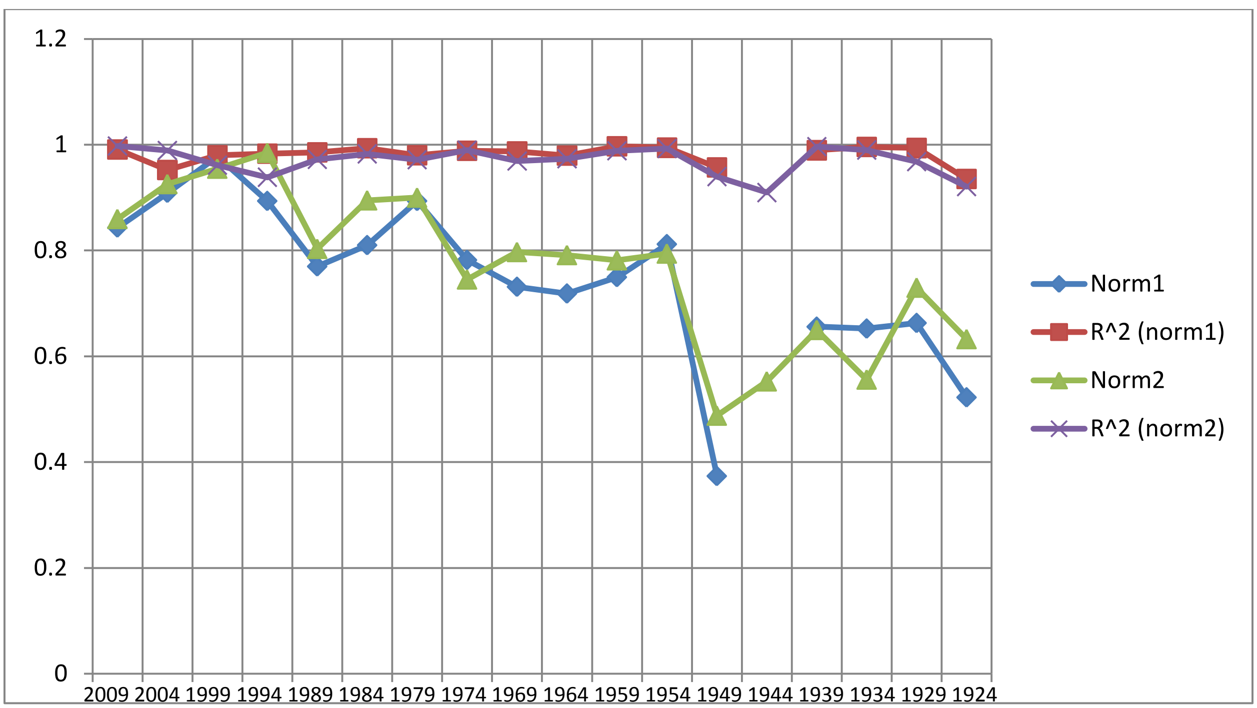
Citing from *t* *(Backwards)*	**Citing to *t* *(Forwards)***	**Norm1**	**R^2**	**Norm2**	**R^2**
2009 – 2000		0.843244	0.990833	0.859476	0.997411
	2000 - 2009	0.912352	0.988674	0.912349	0.988675
2004 - 1995		0.908833	0.95209	0.92551	0.988938
	1995 - 2004	0.954107	0.989244	0.954109	0.989243
1999 - 1990		0.977854	0.980182	0.954596	0.961179
	1990 - 1999	0.878639	0.973891	0.878641	0.97389
1994 - 1985		0.893438	0.982869	0.984524	0.938495
	1985 - 1994	0.809022	0.995177	0.809023	0.995176
1989 - 1980		0.769807	0.985599	0.802998	0.972216
	1980 - 1989	0.846005	0.970092	0.846005	0.970092
1984 - 1975		0.809988	0.993118	0.894706	0.982658
	1975 - 1984	0.802877	0.988983	0.802877	0.988983
1979 - 1970		0.893424	0.980122	0.899878	0.971563
	1970 - 1979	0.794354	0.994586	0.794354	0.994586
1974 - 1965		0.78193	0.988547	0.745188	0.989449
	1965 - 1974	0.669217	0.996255	0.669216	0.996255
1969 - 1960		0.731294	0.987344	0.796999	0.969078
	1960 – 1969	0.701806	0.998814	0.701806	0.998814
1964 - 1955		0.718588	0.979526	0.79126	0.973439
	1955 – 1964	0.718787	0.995721	0.718787	0.995721
1959 - 1950		0.749621	0.997064	0.781369	0.988465
	1950 - 1959	0.698781	0.980409	0.698782	0.980409
1954 - 1945		0.811827	0.994271	0.79409	0.992803
	1945 – 1954	0.64169	0.990599	0.64169	0.990599
1949 - 1940		0.373865	0.956877	0.487689	0.939714
	1940 – 1949	0.956531	0.882359	0.956531	0.882359
1944 - 1935		Undetermined	Undetermined	0.552488	0.909524
	1935 - 1944	0.501727	0.974634	0.501727	0.974634
1939 - 1930		0.655915	0.989402	0.649698	0.996179
	1930 – 1939	0.635139	0.987513	0.635139	0.987513
1934 - 1925		0.652382	0.99567	0.555887	0.989898
	1925 - 1934	0.674329	0.989638	0.674329	0.989638
1929 - 1920		0.662994	0.993686	0.72963	0.967912
	1920 - 1929	0.053763	0.698353	0.053764	0.698353
1924 - 1915		0.522511	0.935125	0.632362	0.920922
	1915 – 1924	0.003953	0.564755	0.003953	0.564755

Citing from *t (Backwards)*

Note that:

2009 - 2000 means papers published in 2009 citing to papers published in 2008, 2007, 2006, …, 2000

Time interval	Norm1	R^2 (norm1)	Norm2	R^2 (norm2)
2009 – 2000	0.843244	0.990833	0.859476	0.997411
2004 – 1995	0.908833	0.95209	0.92551	0.988938
1999 – 1990	0.977854	0.980182	0.954596	0.961179
1994 – 1985	0.893438	0.982869	0.984524	0.938495
1989 – 1980	0.769807	0.985599	0.802998	0.972216
1984 – 1975	0.809988	0.993118	0.894706	0.982658
1979 – 1970	0.893424	0.980122	0.899878	0.971563
1974 – 1965	0.78193	0.988547	0.745188	0.989449
1969 – 1960	0.731294	0.987344	0.796999	0.969078
1964 – 1955	0.718588	0.979526	0.79126	0.973439
1959 – 1950	0.749621	0.997064	0.781369	0.988465
1954 – 1945	0.811827	0.994271	0.79409	0.992803
1949 – 1940	0.373865	0.956877	0.487689	0.939714
1944 – 1935	Not expo	Not expo	0.552488	0.909524
1939 – 1930	0.655915	0.989402	0.649698	0.996179
1934 – 1925	0.652382	0.99567	0.555887	0.989898
1929 – 1920	0.662994	0.993686	0.72963	0.967912
1924 – 1915	0.522511	0.935125	0.632362	0.920922

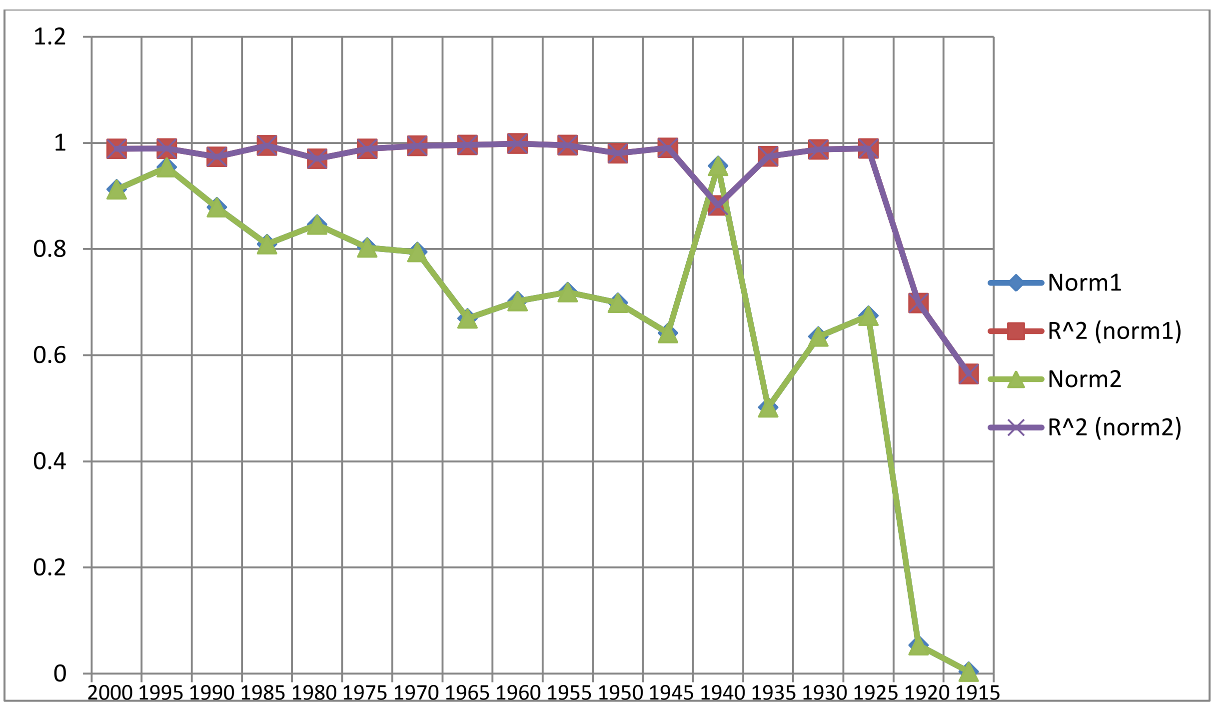


Citing to *t* (Forwards)

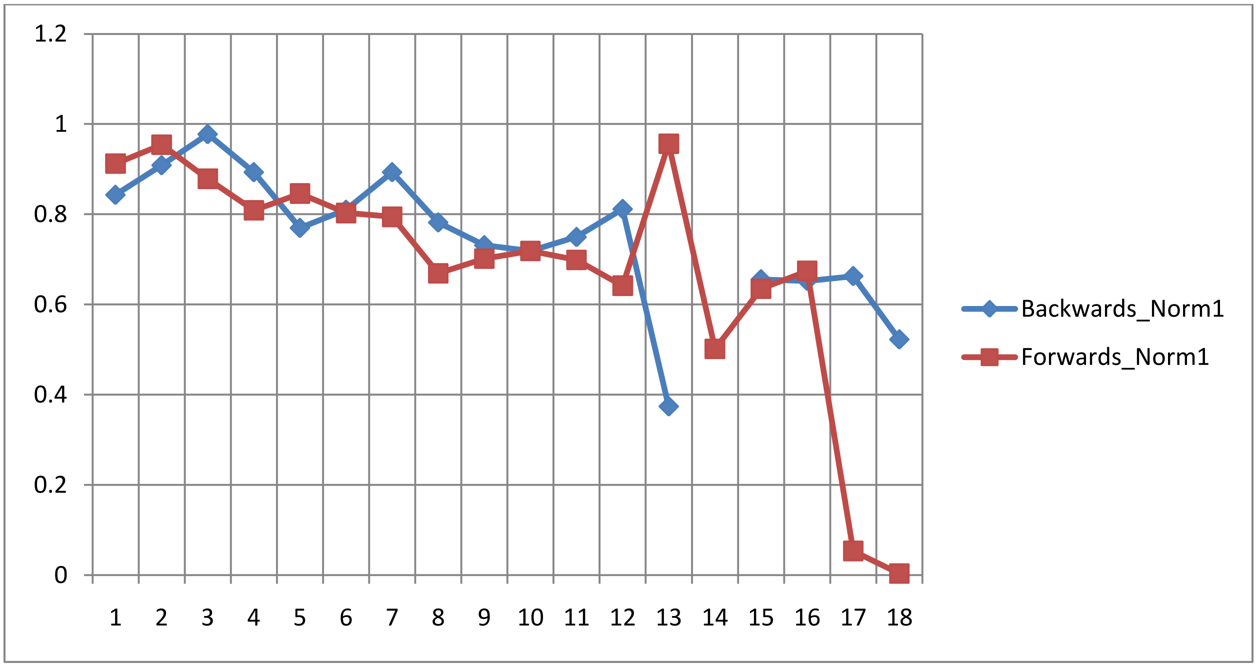
Note that:

2000 - 2009 means papers published in 2000 are cited by papers published in 2001, 2002, 2003, …, 2009

Time interval	Norm1	R^2 (norm1)	Norm2	R^2 (norm2)
2000 – 2009	0.912352	0.988674	0.912349	0.988675
1995 – 2004	0.954107	0.989244	0.954109	0.989243
1990 – 1999	0.878639	0.973891	0.878641	0.97389
1985 – 1994	0.809022	0.995177	0.809023	0.995176
1980 – 1989	0.846005	0.970092	0.846005	0.970092
1975 – 1984	0.802877	0.988983	0.802877	0.988983
1970 – 1979	0.794354	0.994586	0.794354	0.994586
1965 – 1974	0.669217	0.996255	0.669216	0.996255
1960 – 1969	0.701806	0.998814	0.701806	0.998814
1955 – 1964	0.718787	0.995721	0.718787	0.995721
1950 – 1959	0.698781	0.980409	0.698782	0.980409
1945 – 1954	0.64169	0.990599	0.64169	0.990599
1940 – 1949	0.956531	0.882359	0.956531	0.882359
1935 – 1944	0.501727	0.974634	0.501727	0.974634
1930 – 1939	0.635139	0.987513	0.635139	0.987513
1925 – 1934	0.674329	0.989638	0.674329	0.989638
1920 – 1929	0.053763	0.698353	0.053764	0.698353
1915 – 1924	0.003953	0.564755	0.003953	0.564755

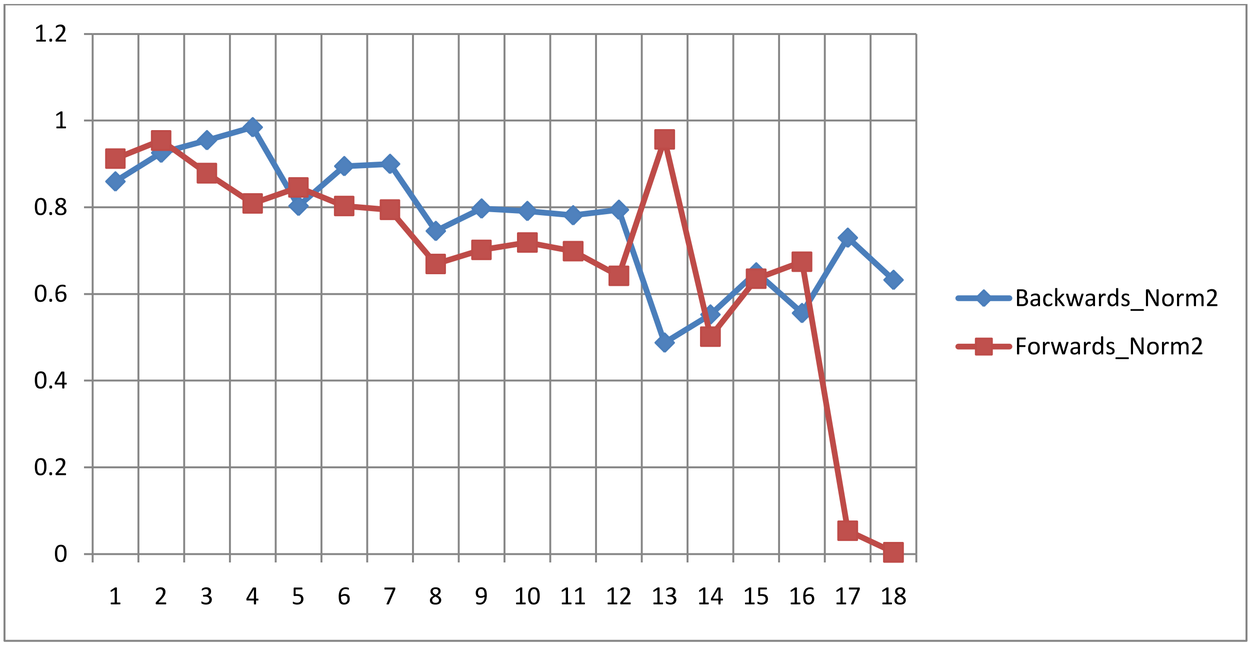


Compare Norm1



x	interval	Backwards_Norm1	Forwards_Norm1
1	2000 – 2009	0.843244	0.912352
2	1995 – 2004	0.908833	0.954107
3	1990 – 1999	0.977854	0.878639
4	1985 – 1994	0.893438	0.809022
5	1980 – 1989	0.769807	0.846005
6	1975 – 1984	0.809988	0.802877
7	1970 – 1979	0.893424	0.794354
8	1965 – 1974	0.78193	0.669217
9	1960 – 1969	0.731294	0.701806
10	1955 – 1964	0.718588	0.718787
11	1950 – 1959	0.749621	0.698781
12	1945 – 1954	0.811827	0.64169
13	1940 – 1949	0.373865	0.956531
14	1935 – 1944		0.501727
15	1930 – 1939	0.655915	0.635139
16	1925 – 1934	0.652382	0.674329
17	1920 – 1929	0.662994	0.053763
18	1915 – 1924	0.522511	0.003953

Compare Norm2

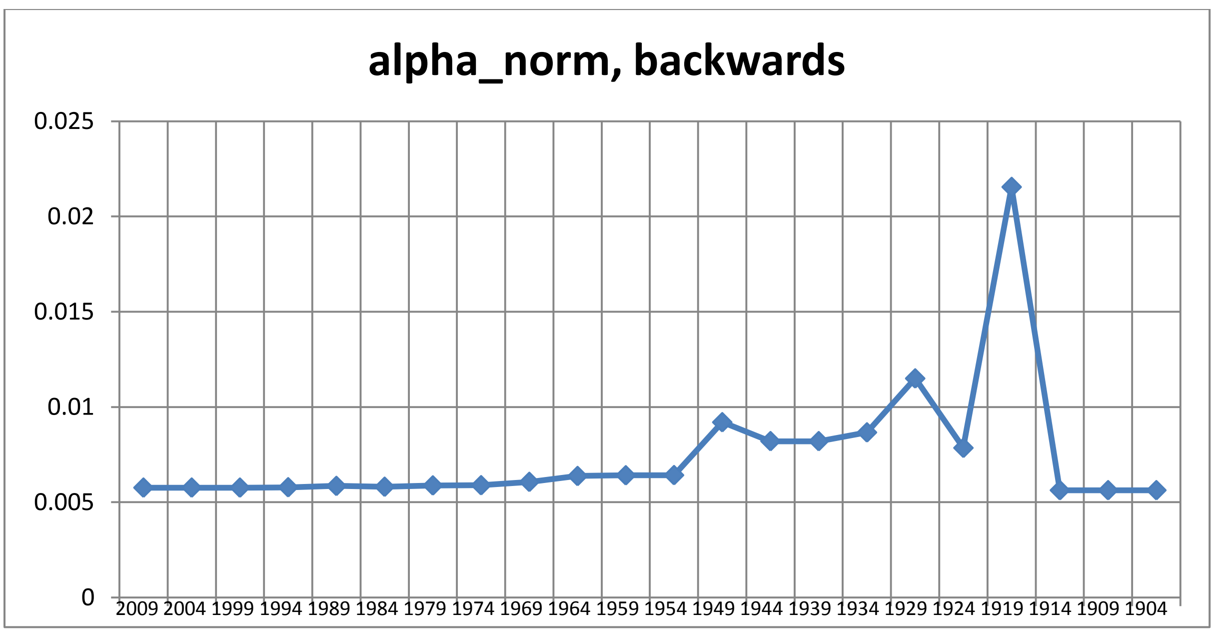


x	interval	Backwards_Norm2	Forwards_Norm2
1	2000 – 2009	0.859476	0.912349
2	1995 – 2004	0.925510	0.954109
3	1990 – 1999	0.954596	0.878641
4	1985 – 1994	0.984524	0.809023
5	1980 – 1989	0.802998	0.846005
6	1975 – 1984	0.894706	0.802877
7	1970 – 1979	0.899878	0.794354
8	1965 – 1974	0.745188	0.669216
9	1960 – 1969	0.796999	0.701806
10	1955 – 1964	0.791260	0.718787
11	1950 – 1959	0.781369	0.698782
12	1945 – 1954	0.794090	0.641690
13	1940 – 1949	0.487689	0.956531
14	1935 – 1944	0.552488	0.501727
15	1930 – 1939	0.649698	0.635139
16	1925 – 1934	0.555887	0.674329
17	1920 – 1929	0.729630	0.053764
18	1915 – 1924	0.632362	0.003953

Alpha estimation

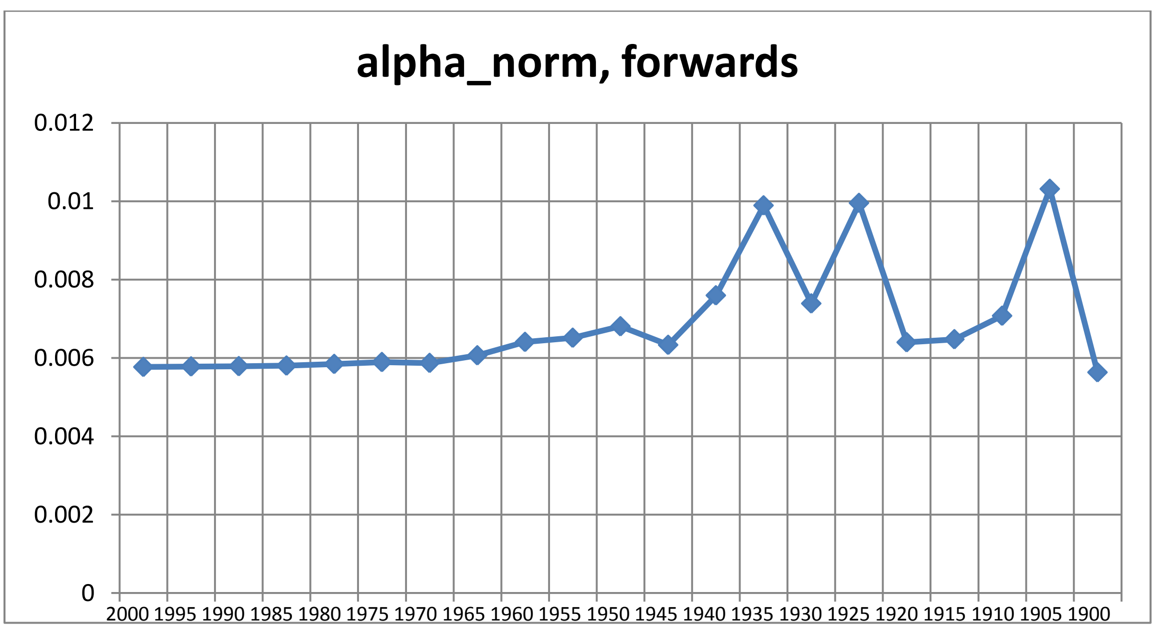
Backwards

Time interval	alpha_norm	1/(1-alpha)	R^2	max(length)
2009 – 2000	0.005768	1.005801	0.999969	9
2004 – 1995	0.00577	1.005804	0.999969	9
1999 – 1990	0.005769	1.005802	0.999969	9
1994 – 1985	0.005787	1.005821	0.999969	9
1989 – 1980	0.005856	1.005891	0.999971	9
1984 – 1975	0.005813	1.005847	0.99997	9
1979 – 1970	0.005884	1.005918	0.999971	9
1974 – 1965	0.005893	1.005928	0.999971	9
1969 – 1960	0.006069	1.006106	0.999972	9
1964 – 1955	0.006376	1.006417	0.999975	9
1959 – 1950	0.006415	1.006456	0.999976	9
1954 – 1945	0.006424	1.006465	0.999976	9
1949 – 1940	0.0092	1.009285	0.999989	9
1944 – 1935	0.008206	1.008274	0.999985	9
1939 – 1930	0.008206	1.008274	0.999985	9
1934 – 1925	0.008668	1.008743	0.999987	9
1929 – 1920	0.0115	1.011634	0.999993	6
1924 – 1915	0.007855	1.007917	0.999984	3
1919 – 1910	0.021548	1.022023	0.999999	3
1914 – 1905	0.005633	1.005665	0.999969	1
1909 – 1900	0.005633	1.005665	0.999969	1
1904 – 1895	0.005633	1.005665	0.999969	1

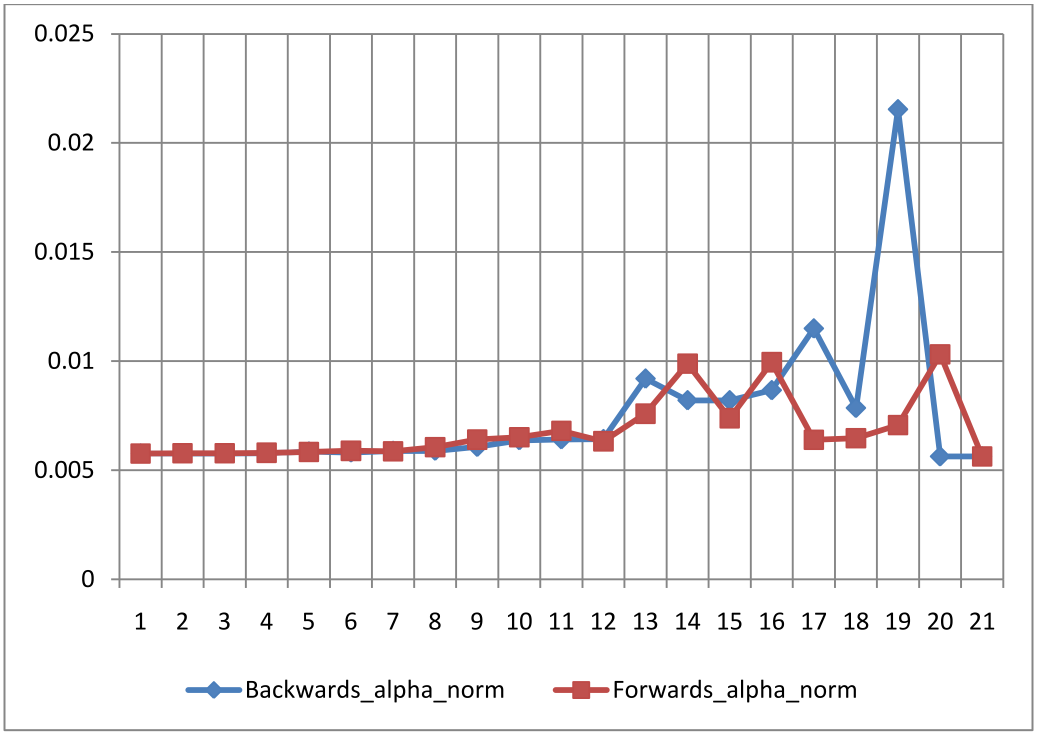


Forwards

Time interval	alpha_norm	1/(1-alpha)	R^2	max(length)
2000 – 2009	0.005769	1.005802	0.999969	9
1995 – 2004	0.005782	1.005815	0.999969	9
1990 – 1999	0.005788	1.005822	0.999969	9
1985 – 1994	0.005803	1.005837	0.99997	9
1980 – 1989	0.005845	1.005879	0.99997	9
1975 – 1984	0.005894	1.005929	0.999971	9
1970 – 1979	0.005871	1.005906	0.999971	9
1965 – 1974	0.006065	1.006102	0.999972	9
1960 – 1969	0.006407	1.006448	0.999975	9
1955 – 1964	0.006514	1.006557	0.999977	9
1950 – 1959	0.006806	1.006853	0.999979	9
1945 – 1954	0.006332	1.006373	0.999974	9
1940 – 1949	0.007595	1.007653	0.999984	9
1935 – 1944	0.00989	1.009989	0.999991	9
1930 – 1939	0.007389	1.007444	0.999982	9
1925 – 1934	0.009953	1.010053	0.999991	9
1920 – 1929	0.006398	1.006439	0.999975	3
1915 – 1924	0.006476	1.006518	0.999976	2
1910 – 1919	0.007075	1.007125	0.99998	2
1905 – 1914	0.010313	1.01042	0.999991	2
1900 – 1909	0.005633	1.005665	0.999969	1
1985 – 1904				0



Compare forwards and backwards



x	Time interval	Backwards_alpha_norm	Forwards_alpha_norm
1	2000 – 2009	0.005768	0.005769
2	1995 – 2004	0.00577	0.005782
3	1990 – 1999	0.005769	0.005788
4	1985 – 1994	0.005787	0.005803
5	1980 – 1989	0.005856	0.005845
6	1975 – 1984	0.005813	0.005894
7	1970 – 1979	0.005884	0.005871
8	1965 – 1974	0.005893	0.006065
9	1960 – 1969	0.006069	0.006407
10	1955 – 1964	0.006376	0.006514
11	1950 – 1959	0.006415	0.006806
12	1945 – 1954	0.006424	0.006332
13	1940 – 1949	0.0092	0.007595
14	1935 – 1944	0.008206	0.00989
15	1930 – 1939	0.008206	0.007389
16	1925 – 1934	0.008668	0.009953
17	1920 – 1929	0.0115	0.006398
18	1915 – 1924	0.007855	0.006476
19	1910 – 1919	0.021548	0.007075
20	1905 – 1914	0.005633	0.010313
21	1900 – 1909	0.005633	0.005633